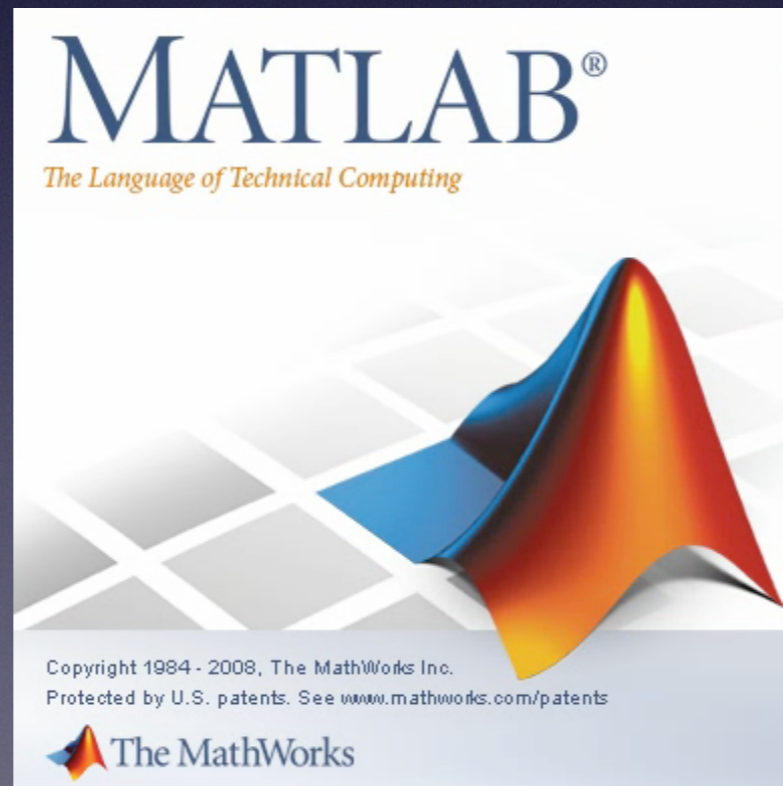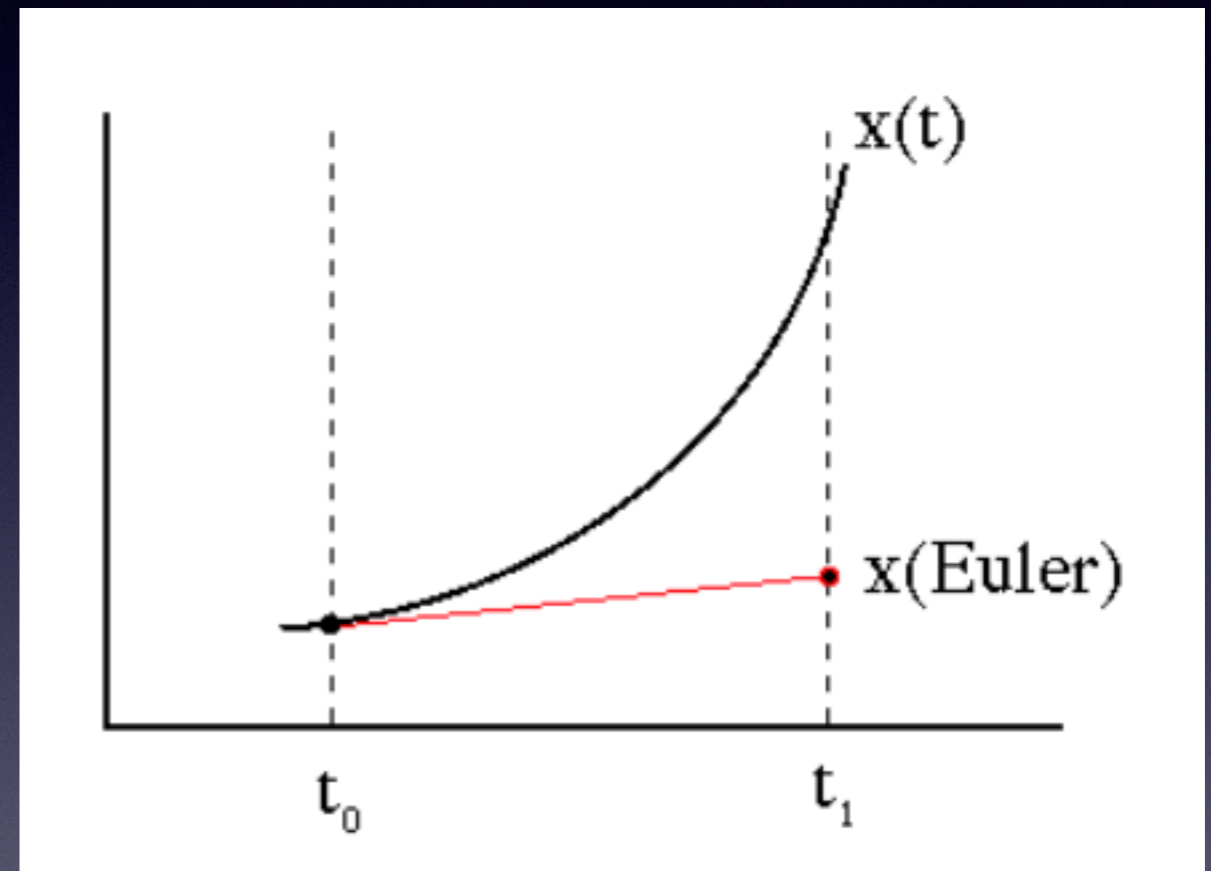# MATLAB

▸ Numerical Integration with ode45

from Euler to
Runge-Kutta

Numerical Integration

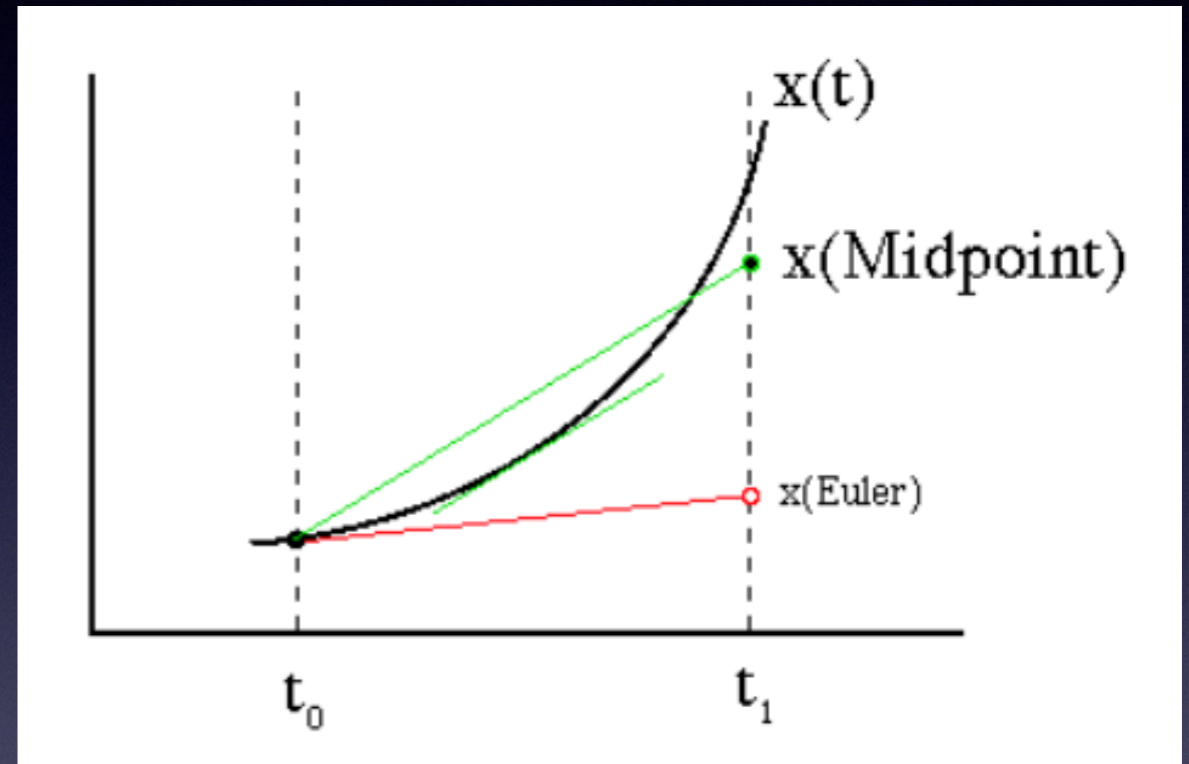Euler Method

$$x(t + \Delta t) = x(t) + f(x(t), t) \Delta t$$

Numerical Integration

Midpoint Method

$$x_{n+1} = x_n + f\left(x_n + f(x_n, t_n)\frac{1}{2}\Delta t, t_n + \frac{1}{2}\Delta t\right)\Delta t$$
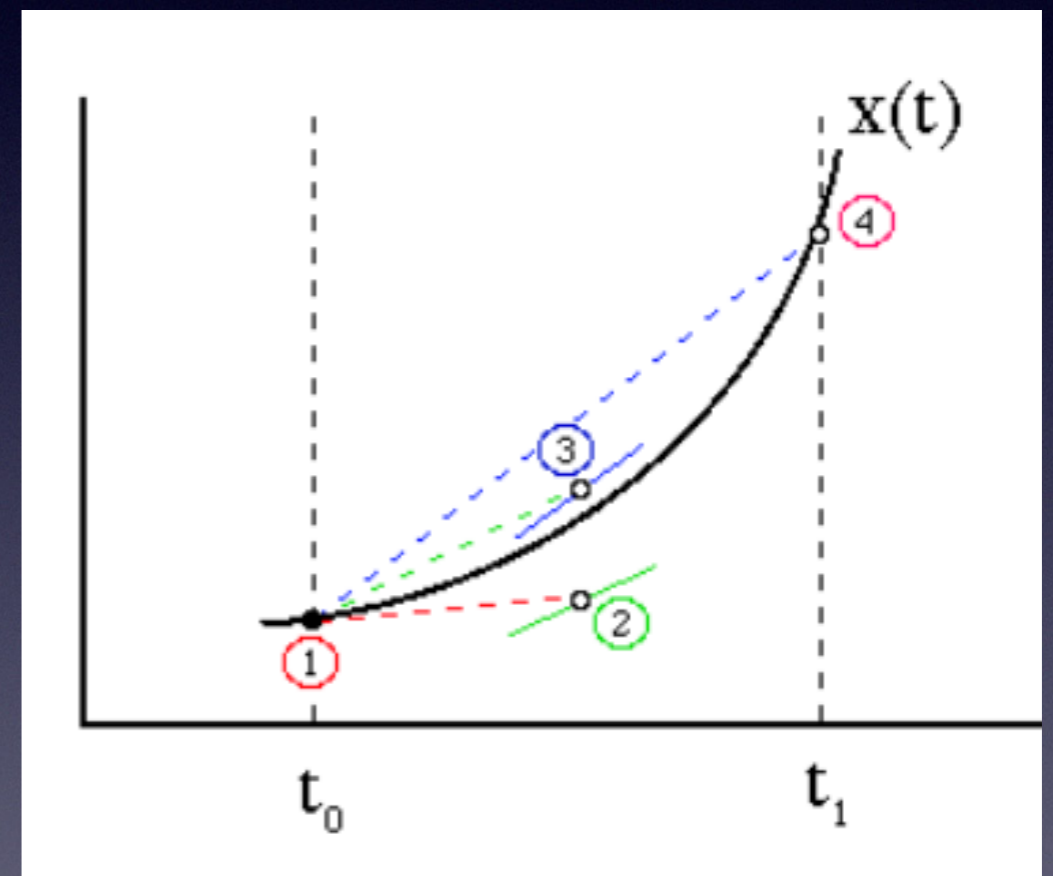
Numerical Integration

RungeKutta4 Method

$$k_1 = f(x_n, t_n)\Delta t$$

$$k_2 = f(x_n + k_1/2, t_n + \Delta t/2)\Delta t$$

$$k_3 = f(x_n + k_2/2, t_n + \Delta t/2)\Delta t$$

$$k_4 = f(x_n + k_3, t_n + \Delta t)\Delta t$$

$$x_{n+1} = x_n + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} + \mathcal{O}\left((\Delta t)^5\right)$$

4th order Taylor expansion